Electrical Circuits (2)

Lecture 1
Intro. & Review

Dr.Eng. Basem ElHalawany

Course Info

Title	Electric Circuits (2)
Lecturer:	Dr. Basem ElHalawany
Lecturer Webpage:	http://www.bu.edu.eg/staff/basem.mamdoh
Teaching Assistant (TA)	Eng. Asmaa yehia
Course Webpage	http://www.bu.edu.eg/staff/basem.mamdoh-courses/13717
References	Multiple references will be used
Software Packages	Proteus Design Suite
Assessment 75/50	1. Final Term Exam (75) 2. Mid Term Exam 3. Proteus Simulation and/or Hardware Implementation 4. Reports

References

A. Circuit Analysis – Theories and Practice (Robinson & Miller)

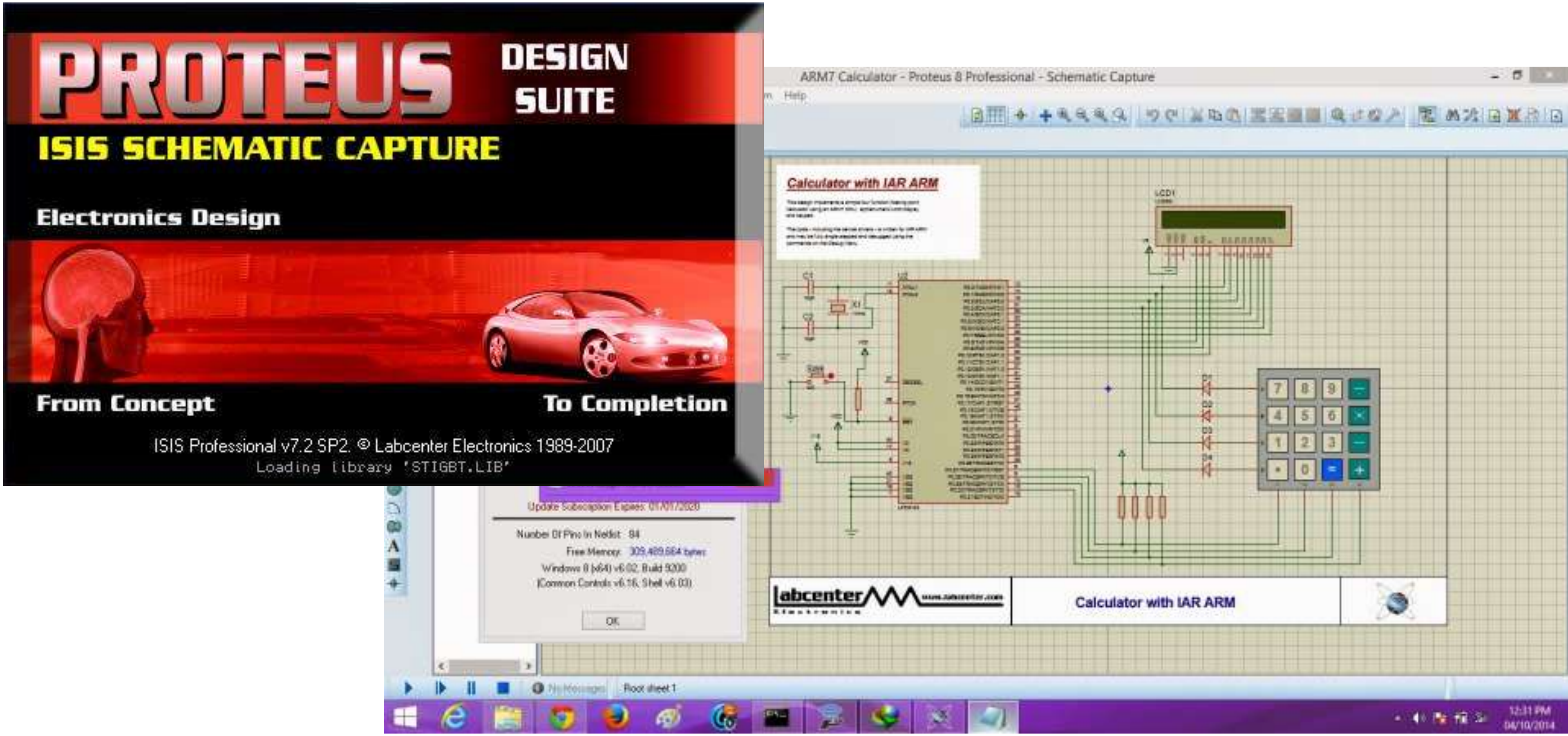
B. Fundamentals of Electric Circuits (Alexander and Sadiku)

C. Principles of Electric Circuits (Floyd)

Main Topics

1. Resonance
2. Magnetically Coupled Circuits
3. Three-Phase Circuits
4. Transient Analysis

Proteus Design Suite



Check the course website for Download and Installation details

Links for Software tutorials are added to the URL section

Review

Ch (17) : ac Series-Parallel Circuits

- The rules and laws which were developed for dc circuits will apply equally well for ac circuits.

  - ✓ Ohm's law,
  - ✓ The voltage divider rule,
  - ✓ Kirchhoff's voltage law,
  - ✓ Kirchhoff's current law, and
  - ✓ The current divider rule.

- The major difference between solving dc and ac circuits is that analysis of ac circuits requires using vector algebra.

- **you should be able to add and subtract any number of vector quantities.**

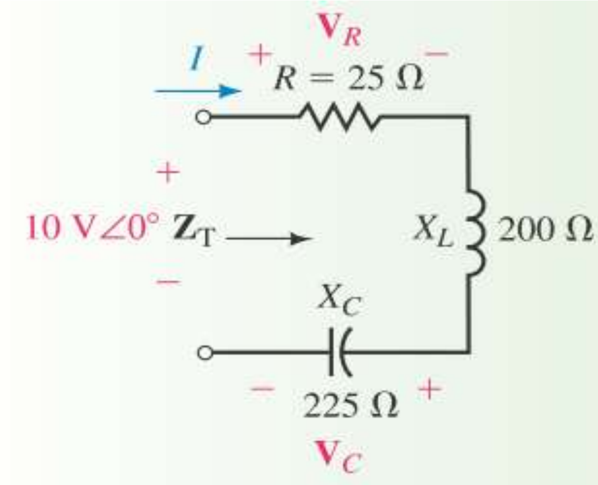
EXAMPLE 18–5

Consider the network of Figure 18–20.

a. Find $\mathbf{Z}_T$.

b. Sketch the impedance diagram for the network and indicate whether the total impedance of the circuit is inductive, capacitive, or resistive.

c. Use Ohm's law to determine $\mathbf{I}$, $\mathbf{V}_R$, and $\mathbf{V}_C$.

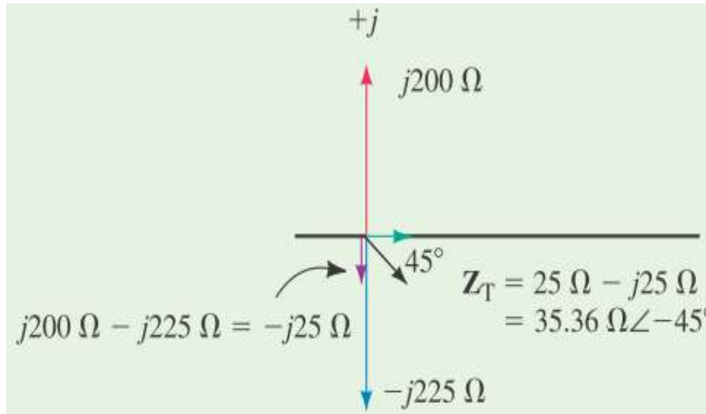


Solution

a. The total impedance is the vector sum

$$\mathbf{Z}_T = 25\ \Omega + j200\ \Omega + (-j225\ \Omega)$$
$$= 25\ \Omega - j25\ \Omega$$
$$= 35.36\ \Omega\angle -45°$$

b. The corresponding impedance diagram is shown in Figure 18–21.



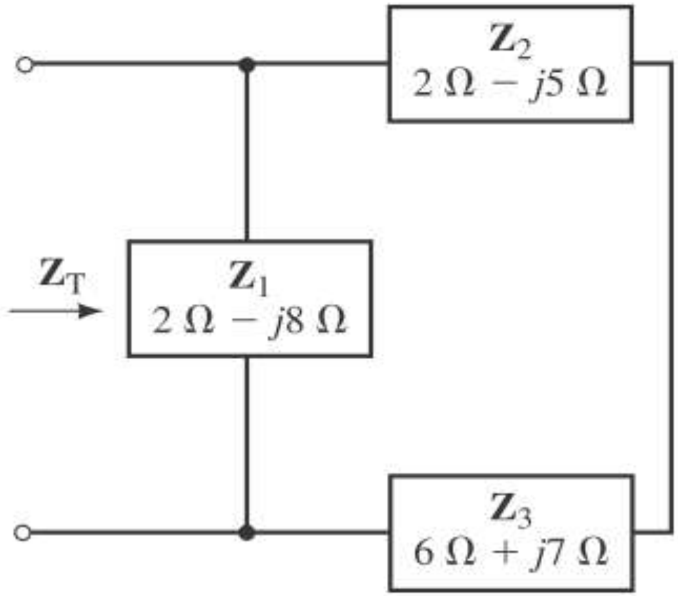
Because the total impedance has a negative reactance term ($-j25\ \Omega$), $\mathbf{Z}_T$ is capacitive.

c. $$\mathbf{I} = \frac{10\ \text{V}\angle 0°}{35.36\ \Omega\angle -45°} = 0.283\ \text{A}\angle 45°$$

$$\mathbf{V}_R = (282.8\ \text{mA}\angle 45°)(25\ \Omega\angle 0°) = 7.07\ \text{V}\angle 45°$$

$$\mathbf{V}_C = (282.8\ \text{mA}\angle 45°)(225\ \Omega\angle -90°) = 63.6\ \text{V}\angle -45°$$

Series-Parallel Circuits



The total impedance of the network is expressed as

$$\mathbf{Z}_T = \mathbf{Z}_1 \parallel (\mathbf{Z}_2 + \mathbf{Z}_3)$$

$$\begin{aligned}
\mathbf{Z}_T &= (2\ \Omega - j8\ \Omega) \| (2\ \Omega - j5\ \Omega + 6\ \Omega + j7\ \Omega) \\
&= (2\ \Omega - j8\ \Omega) \parallel (8\ \Omega + j2\ \Omega) \\
&= \frac{(2\ \Omega - j8\ \Omega)(8\ \Omega + j2\ \Omega)}{2\ \Omega - j8\ \Omega + 8\ \Omega + j2\ \Omega)} \\
&= \frac{(8.246\ \Omega\angle -75.96^\circ)(8.246\ \Omega\angle 14.04^\circ)}{11.66\ \Omega\angle -30.96^\circ} \\
&= 5.832\ \Omega\angle -30.96^\circ = 5.0\ \Omega - j3.0\ \Omega
\end{aligned}$$

Kirchhoff's Voltage Law and the Voltage Divider Rule

- Kirchhoff's voltage law for ac circuits may be stated as:

> The phasor sum of voltage drops and voltage rises around a closed loop is equal to zero.

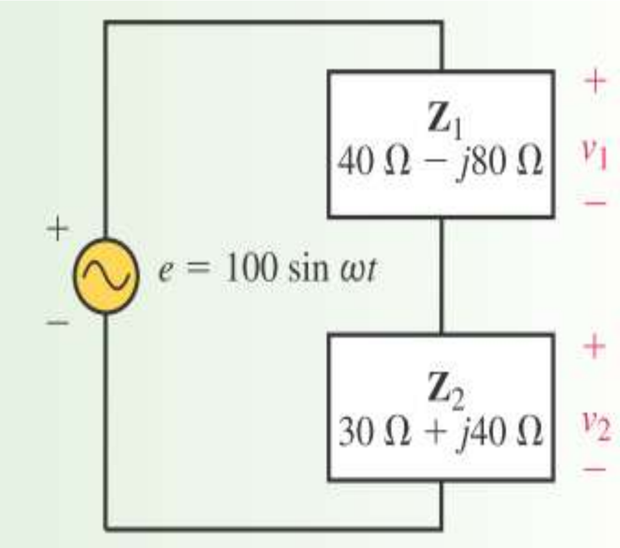
- Remember : The summation is generally done more easily in rectangular form than in the polar form.

EXAMPLE 18–10

Consider the circuit of Figure 18–32:

a. Calculate the sinusoidal voltages v_1 and v_2 using phasors and the voltage divider rule.

b. Sketch the phasor diagram showing **E**, $\mathbf{V}_1$, and $\mathbf{V}_2$.



Solution

a. The phasor form of the voltage source is determined as

$$e = 100 \sin \omega t \Leftrightarrow \mathbf{E} = 70.71 \angle V\, 0°$$

Kirchhoff's Voltage Law and the Voltage Divider Rule

EXAMPLE 18–10

a. The phasor form of the voltage source is determined as

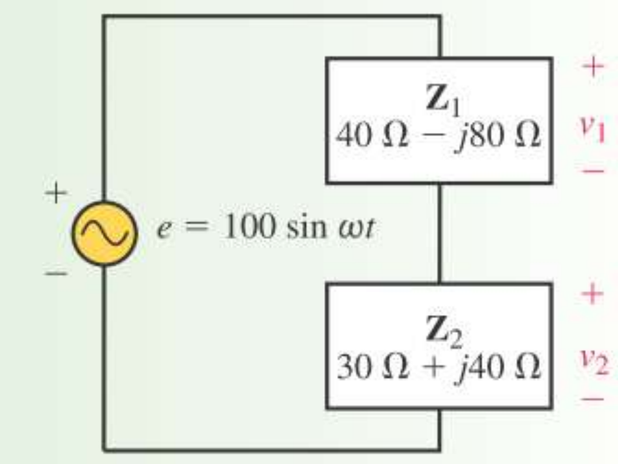
$$e = 100 \sin \omega t \Leftrightarrow \mathbf{E} = 70.71 \angle V\ 0°$$

Applying VDR, we get

$$\mathbf{V}_1 = \left(\frac{40\ \Omega - j80\ \Omega}{(40\ \Omega - j80\ \Omega) + (30\ \Omega + j40\ \Omega)}\right)(70.71\ \text{V}\angle 0°)$$

$$= \left(\frac{89.44\ \Omega\angle -63.43°}{80.62\ \Omega\angle -29.74°}\right)(70.71\ \text{V}\angle 0°)$$

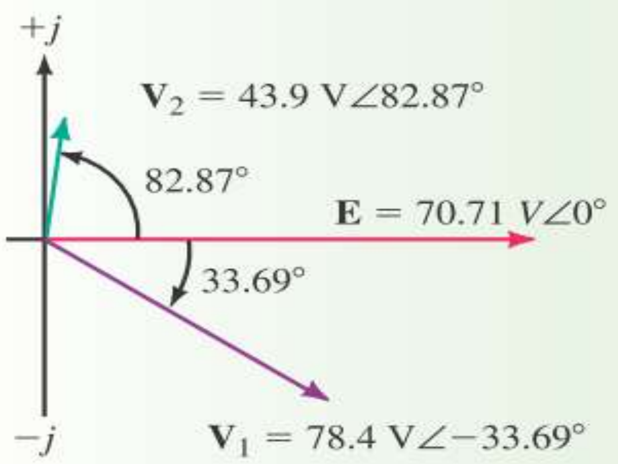
$$= 78.4\ \text{V}\angle -33.69°$$



$$\mathbf{V}_2 = \left(\frac{30\ \Omega + j40\ \Omega}{(40\ \Omega - j80\ \Omega) + (30\ \Omega + j40\ \Omega)}\right)(70.71\ \text{V}\angle 0°)$$

$$= \left(\frac{50.00\ \Omega\angle 53.13°}{80.62\ \Omega\angle -29.74°}\right)(70.71\ \text{V}\angle 0°)$$

$$= 43.9\ \text{V}\angle 82.87°$$



The sinusoidal voltages are determined to be

$$v_1 = (\sqrt{2})(78.4)\sin(\omega t - 33.69°)$$
$$= 111 \sin(\omega t - 33.69°)$$

and

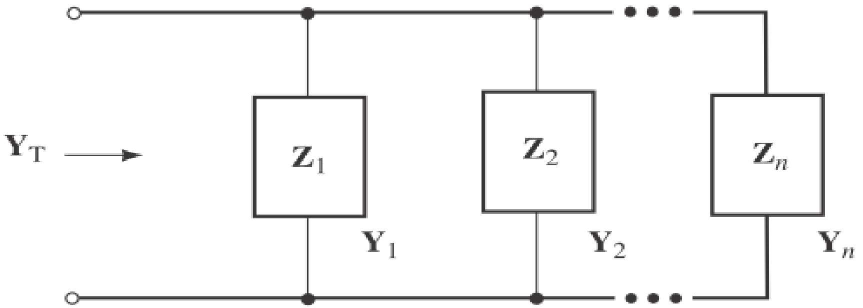
$$v_2 = (\sqrt{2})(43.9)\sin(\omega t + 82.87°)$$
$$= 62.0 \sin(\omega t + 82.87°)$$

ac Parallel Circuits

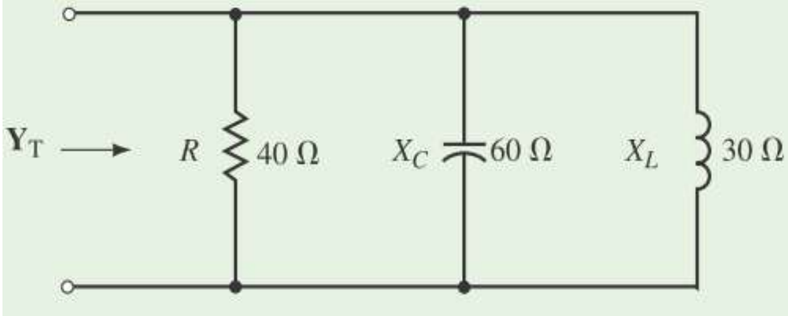
The total admittance is the vector sum of the admittances of the network.

$$\mathbf{Y}_T = \mathbf{Y}_1 + \mathbf{Y}_2 + \cdots + \mathbf{Y}_n \quad (S)$$

$$\mathbf{Z}_T = \frac{1}{\mathbf{Y}_T} = \frac{1}{\mathbf{Y}_1 + \mathbf{Y}_2 + \cdots + \mathbf{Y}_n}$$



Find the equivalent admittance and impedance of the network of Figure 18–38. Sketch the admittance diagram.



Solution The admittances of the various parallel elements are

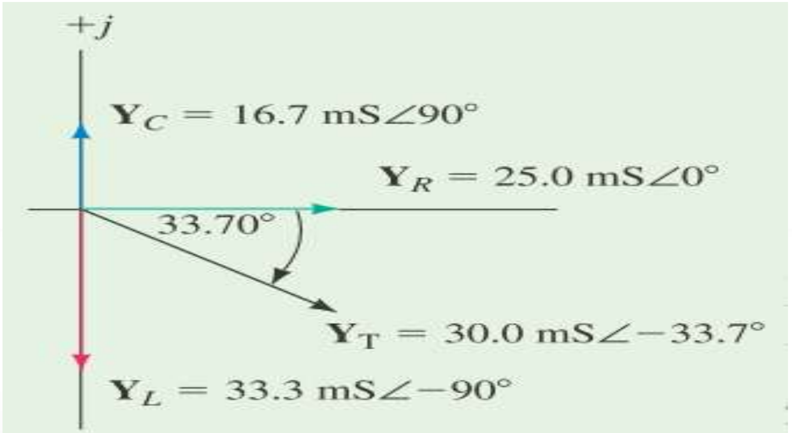
$$\mathbf{Y}_1 = \frac{1}{40\ \Omega\angle 0^\circ} = 25.0\ \text{mS}\angle 0^\circ = 25.0\ \text{mS} + j0$$

$$\mathbf{Y}_2 = \frac{1}{60\ \Omega\angle -90^\circ} = 16.\bar{6}\ \text{mS}\angle 90^\circ = 0 + j16.\bar{6}\ \text{mS}$$

$$\mathbf{Y}_3 = \frac{1}{30\ \Omega\angle 90^\circ} = 33.\bar{3}\ \text{mS}\angle -90^\circ = 0 - j33.\bar{3}\ \text{mS}$$

The total admittance is determined as

$$\begin{aligned}\mathbf{Y}_T &= \mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3 \\ &= 25.0\ \text{mS} + j16.\bar{6}\ \text{mS} + (-j33.\bar{3}\ \text{mS}) \\ &= 25.0\ \text{mS} - j16.\bar{6}\ \text{mS} \\ &= 30.0\ \text{mS}\angle -33.69^\circ\end{aligned}$$



$$\mathbf{Z}_T = \frac{1}{\mathbf{Y}_T} = \frac{1}{30.0\ \text{mS}\angle -33.69^\circ} = 33.3\ \Omega\angle 33.69^\circ$$

Find the input impedance of the circuit in Fig. 9.23. Assume that the circuit operates at $\omega = 50$ rad/s.

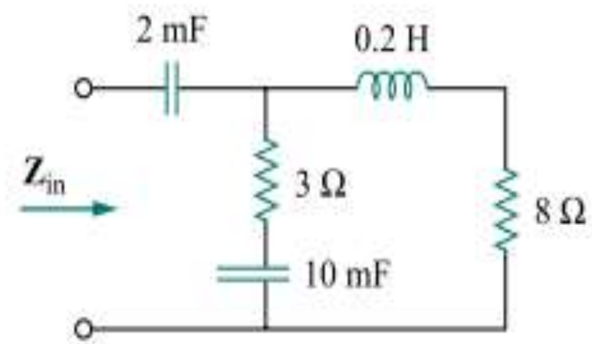


Figure 9.23 For Example 9.10.

Solution:

Let

$\mathbf{Z}_1$ = Impedance of the 2-mF capacitor

$\mathbf{Z}_2$ = Impedance of the 3-Ω resistor in series with the 10-mF capacitor

$\mathbf{Z}_3$ = Impedance of the 0.2-H inductor in series with the 8-Ω resistor

$$\mathbf{Z}_1 = \frac{1}{j\omega C} = \frac{1}{j50 \times 2 \times 10^{-3}} = -j10\ \Omega$$

$$\mathbf{Z}_2 = 3 + \frac{1}{j\omega C} = 3 + \frac{1}{j50 \times 10 \times 10^{-3}} = (3 - j2)\ \Omega$$

$$\mathbf{Z}_3 = 8 + j\omega L = 8 + j50 \times 0.2 = (8 + j10)\ \Omega$$

The input impedance is

$$\mathbf{Z}_{in} = \mathbf{Z}_1 + \mathbf{Z}_2 \parallel \mathbf{Z}_3 = -j10 + \frac{(3 - j2)(8 + j10)}{11 + j8}$$

$$= -j10 + \frac{(44 + j14)(11 - j8)}{11^2 + 8^2} = -j10 + 3.22 - j1.07\ \Omega$$

Thus,

$$\mathbf{Z}_{in} = 3.22 - j11.07\ \Omega$$

Examples

Find current **I** in the circuit in Fig.

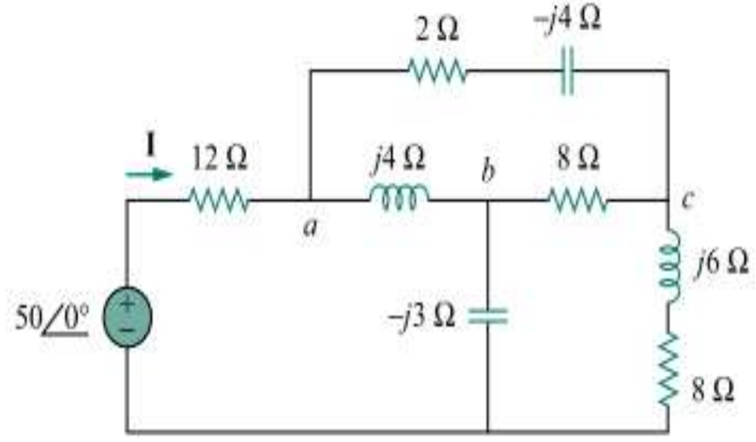


Figure 9.28 For Example 9.12.

Solution:

The delta network connected to nodes *a*, *b*, and *c* can be converted to the *Y* network of Fig. 9.29. We obtain the *Y* impedances as follows using Eq. (9.68):

$$\mathbf{Z}_{an} = \frac{j4(2-j4)}{j4+2-j4+8} = \frac{4(4+j2)}{10} = (1.6+j0.8)\ \Omega$$

$$\mathbf{Z}_{bn} = \frac{j4(8)}{10} = j3.2\ \Omega, \qquad \mathbf{Z}_{cn} = \frac{8(2-j4)}{10} = (1.6-j3.2)\ \Omega$$

The total impedance at the source terminals is

$$\begin{aligned}\mathbf{Z} &= 12 + \mathbf{Z}_{an} + (\mathbf{Z}_{bn} - j3) \parallel (\mathbf{Z}_{cn} + j6 + 8) \\ &= 12 + 1.6 + j0.8 + (j0.2) \parallel (9.6 + j2.8) \\ &= 13.6 + j0.8 + \frac{j0.2(9.6+j2.8)}{9.6+j3} \\ &= 13.6 + j1 = 13.64\underline{/4.204^\circ}\ \Omega\end{aligned}$$

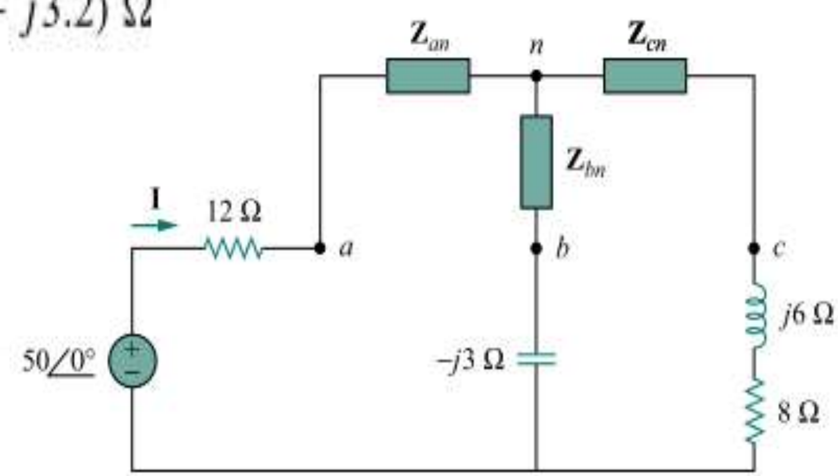


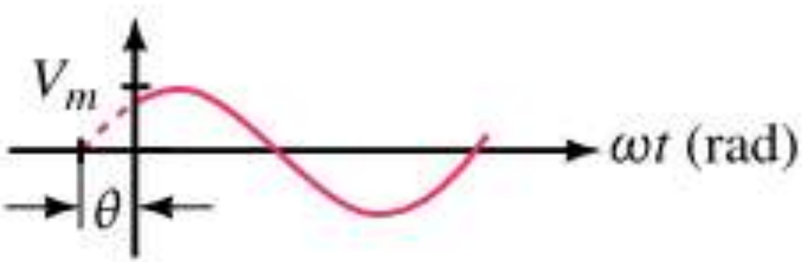
Figure 9.29 The circuit in Fig. 9.28 after delta-to-wye transformation.

The desired current is

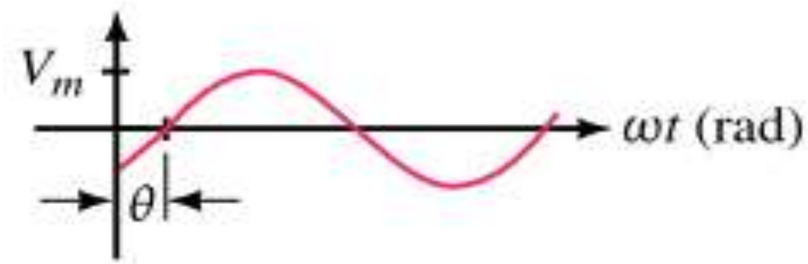
$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{50\underline{/0^\circ}}{13.64\underline{/4.204^\circ}} = 3.666\underline{/-4.204^\circ}\ \text{A}$$

Voltages and Currents with Phase Shifts

- If a sine wave does not pass through zero at t =0 s, it has a phase shift.
- Waveforms may be shifted to the left or to the right



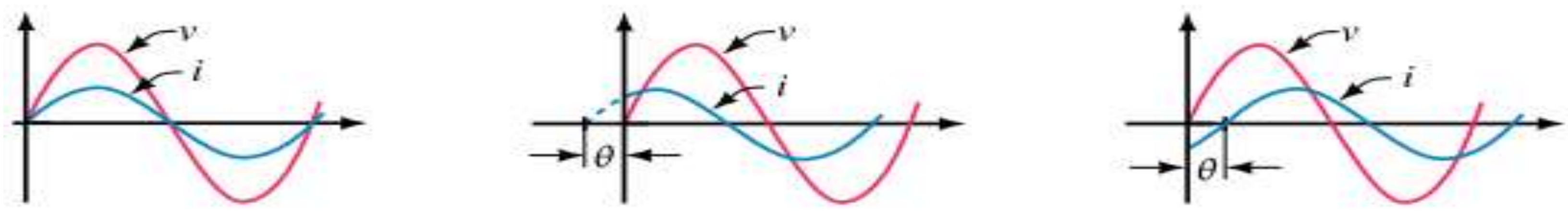
(a) $v = V_m \sin(\omega t + \theta)$



(b) $v = V_m \sin(\omega t - \theta)$

Phasor Difference

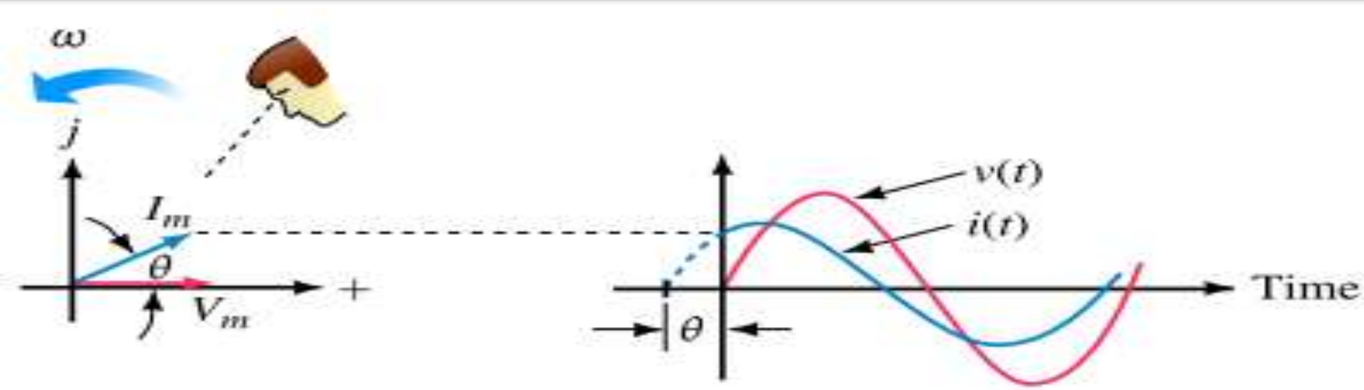
- Phase difference refers to the angular displacement between different waveforms of the same frequency.



(a) In phase (b) Current leads (c) Current lags

FIGURE 15–40 Illustrating phase difference. In these examples, voltage is taken as reference.

- The terms lead and lag can be understood in terms of phasors. If you observe phasors rotating as in Figure, the one that you see passing first is leading and the other is lagging.



(a) I_m leads V_m (b) Therefore, $i(t)$ leads $v(t)$

AC Waveforms and Average Value

- Since ac quantities constantly change its value, we need one single numerical value that truly represents a waveform over its complete cycle.

- Average values are also called **dc values**, because **dc meters** indicate average values rather than instantaneous values.

Average in Terms of the Area Under a Curve:

$$\text{average} = \frac{\text{area under curve}}{\text{length of base}}$$

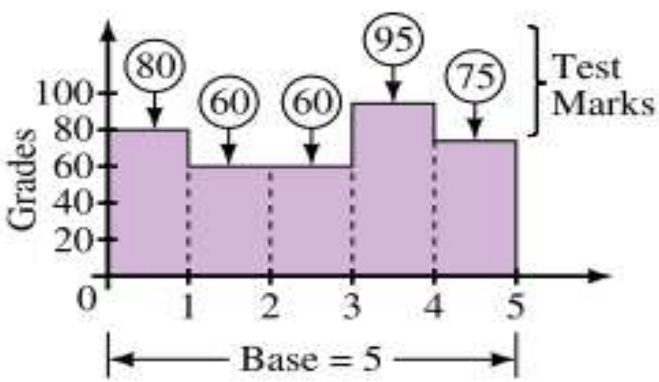


FIGURE 15–50 Determining average by area.

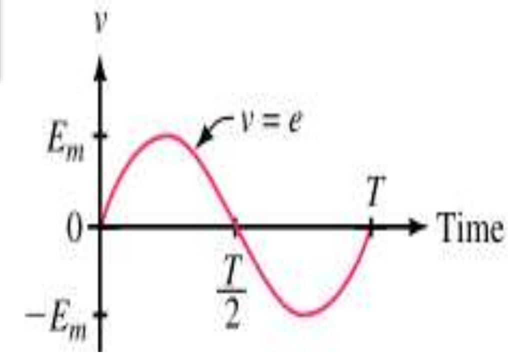
- This approach is valid regardless of waveshape.

$$\text{average} = (80 + 60 + 60 + 95 + 75)/5 = 74$$

Or use area

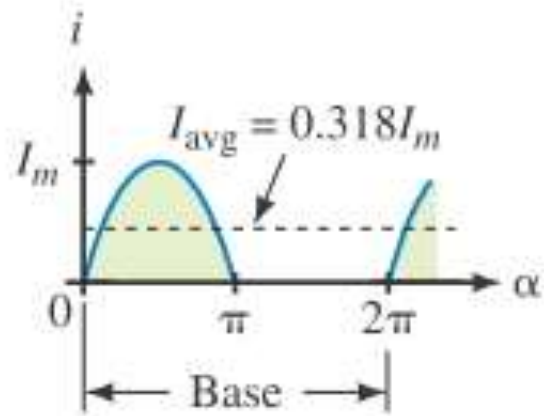
$$\frac{(80 \times 1) + (60 \times 2) + (95 \times 1) + (75 \times 1)}{5} = 74$$

Sine-wave Averages



Full Cycle Sine Wave Average:

- Because a sine wave is symmetrical, its area below the horizontal axis is the same as its area above the axis;
- Thus, over a full cycle its net area is zero, independent of frequency and phase angle.

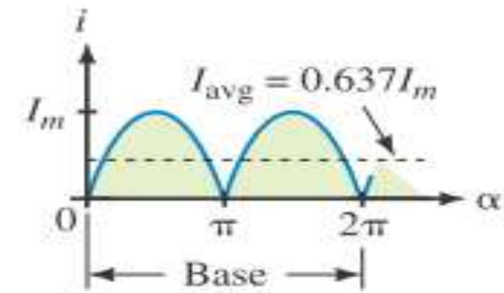


Half-wave average:

- The area under the half-cycle is:

$$\text{area} = \int_0^{\pi} I_m \sin\alpha \, d\alpha = \Big[-I_m \cos\alpha\Big]_0^{\pi} = 2I_m$$

$$I_{avg} = \frac{2I_m}{2\,\pi} = \frac{I_m}{\pi} = 0.318I_m$$



Full-wave average:

$$I_{avg} = \frac{2(2I_m)}{2\,\pi} = \frac{2I_m}{\pi} = 0.637I_m$$

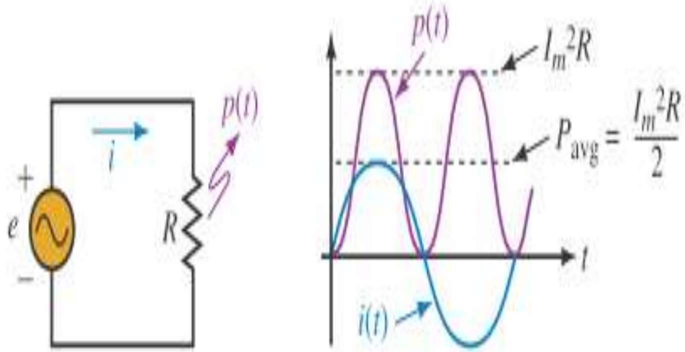
Effective Values - Root Mean Square (rms) Values

- An **effective (rms) value** is an equivalent dc value:
  - ✓ it tells you how many volts of dc that a time-varying waveform is equal to in terms of its ability to produce average power.

For the Sinsusoidal ac case:

$$p(t) = i^2R = (I_m \sin \omega t)^2 R = I_m{}^2 R \sin^2 \omega t = I_m{}^2 R\left[\frac{1}{2}(1 - \cos 2\omega t)\right]$$

$$p(t) = \frac{I_m{}^2 R}{2} - \frac{I_m{}^2 R}{2} \cos 2\omega t$$



$p(t) = i^2R$. Therefore, $p(t)$ varies cyclically.

(a) ac Circuit

Calculating the ac average power:

$$P_{avg} = \text{average of } p(t) = \frac{I_m{}^2 R}{2}$$ **1**

By Equating 1 to achieve the same average power as the dc , we get:

$$P_{avg} = P = I^2 R$$ **2**

$$I^2 = \frac{I_m{}^2}{2} \qquad I = \sqrt{\frac{I_m{}^2}{2}} = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

Which is the rms value

Effective Values - Root Mean Square (rms) Values

Effective voltage can be expressed also as:

$$V_{\text{eff}} = \frac{V_m}{\sqrt{2}} = 0.707V_m$$

effective values for sinusoidal waveforms depend only on amplitude

It is important to note that these relationships hold only for sinusoidal waveforms. However, the concept of effective value applies to all waveforms

General Equation for Effective Values:

$$I_{\text{eff}} = \sqrt{\frac{1}{T}\int_0^T i^2 dt}$$

$$I_{\text{eff}} = \sqrt{\frac{\text{area under the } i^2 \text{ curve}}{\text{base}}}$$

Get the square root of the mean value of the squared waveform.

root - mean - square

Ac Power

In the circuit of Fig. 11.26, $\mathbf{Z}_1 = 60\angle -30^\circ\ \Omega$ and $\mathbf{Z}_2 = 40\angle 45^\circ\ \Omega$. Calculate the total: (a) apparent power, (b) real power, (c) reactive power, and (d) pf.

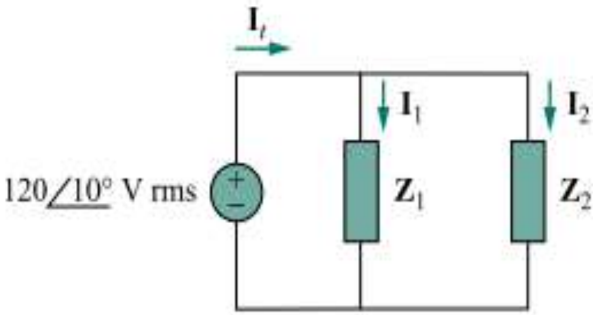


Figure 11.26 For Example 11.14.

Solution:

The current through $\mathbf{Z}_1$ is

$$\mathbf{I}_1 = \frac{\mathbf{V}}{\mathbf{Z}_1} = \frac{120\angle 10^\circ}{60\angle -30^\circ} = 2\angle 40^\circ \text{ A rms}$$

while the current through $\mathbf{Z}_2$ is

$$\mathbf{I}_2 = \frac{\mathbf{V}}{\mathbf{Z}_2} = \frac{120\angle 10^\circ}{40\angle 45^\circ} = 3\angle -35^\circ \text{ A rms}$$

The complex powers absorbed by the impedances are

$$\mathbf{S}_1 = \frac{V_{\text{rms}}^2}{\mathbf{Z}_1^*} = \frac{(120)^2}{60\angle 30^\circ} = 240\angle -30^\circ = 207.85 - j120 \text{ VA}$$

$$\mathbf{S}_2 = \frac{V_{\text{rms}}^2}{\mathbf{Z}_2^*} = \frac{(120)^2}{40\angle -45^\circ} = 360\angle 45^\circ = 254.6 + j254.6 \text{ VA}$$

The total complex power is

$$\mathbf{S}_t = \mathbf{S}_1 + \mathbf{S}_2 = 462.4 + j134.6 \text{ VA}$$

(a) The total apparent power is

$$|\mathbf{S}_t| = \sqrt{462.4^2 + 134.6^2} = 481.6 \text{ VA}.$$

(b) The total real power is

$$P_t = \text{Re}(\mathbf{S}_t) = 462.4 \text{ W or } P_t = P_1 + P_2.$$

(c) The total reactive power is

$$Q_t = \text{Im}(\mathbf{S}_t) = 134.6 \text{ VAR or } Q_t = Q_1 + Q_2.$$

(d) The pf $= P_t/|\mathbf{S}_t| = 462.4/481.6 = 0.96$ (lagging).

Ac Power

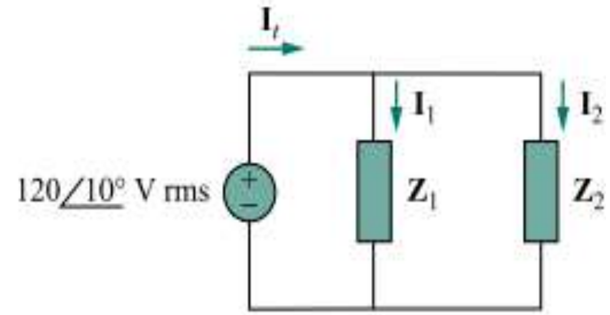


Figure 11.26 For Example 11.14.

(a) The total apparent power is

$$|\mathbf{S}_t| = \sqrt{462.4^2 + 134.6^2} = 481.6 \text{ VA}.$$

(b) The total real power is

$$P_t = \text{Re}(\mathbf{S}_t) = 462.4 \text{ W or } P_t = P_1 + P_2.$$

(c) The total reactive power is

$$Q_t = \text{Im}(\mathbf{S}_t) = 134.6 \text{ VAR or } Q_t = Q_1 + Q_2.$$

(d) The pf $= P_t/|\mathbf{S}_t| = 462.4/481.6 = 0.96$ (lagging).

We may cross check the result by finding the complex power $\mathbf{S}_s$ supplied by the source.

$$\mathbf{I}_t = \mathbf{I}_1 + \mathbf{I}_2 = (1.532 + j1.286) + (2.457 - j1.721)$$
$$= 4 - j0.435 = 4.024\angle -6.21^\circ \text{ A rms}$$
$$\mathbf{S}_s = \mathbf{V}\mathbf{I}_t^* = (120\angle 10^\circ)(4.024\angle 6.21^\circ)$$
$$= 482.88\angle 16.21^\circ = 463 + j135 \text{ VA}$$

which is the same as before.